Simultaneous Equations

A Choose the variable you want to keep.

$$3x + 8y = 74$$
$$3x + y = 25$$

The x coefficient (number in front of the x) is the **same** in both cases. You can take one from the other leaving just the y part of the equation. It doesn't matter if the number is negative or positive at this stage.

1 $$6x + 5y = 74$$
$$6x + 2y = 25$$

2 $$x + 4y = 20$$
$$3x + 4y = 36$$

3 $$5x + y = 37$$
$$3x + y = 23$$

4 $$9x + 3y = 9$$
$$8x - 3y = 25$$

5 $$4x + 3y = 32$$
$$8y - 4x = 24$$

6 $$9x + 5y = 53$$
$$4y - 9x = 10$$

7 $$3(7x + 3y) = 189$$
$$9y - 12x = -9$$

B Add or subtract the equations

$$3x + 8y = 74$$
$$3x + y = 25$$

Here, the "signs" for the coefficients in front of the variable you want to remove are the **same** (ie both positive) so you need to **subtract** the equations.

$$9x + 5y = 53$$
$$4y - 9x = 10$$

Here, the "signs" for the coefficients in front of the variable you want to remove are the **different** (ie positive and negative) so you need to **add** the equations so the x terms cancel each other out.

1 $$6x + 5y = 74$$
$$6x + 2y = 25$$

2 $$x - 4y = 2$$
$$3x - 4y = 22$$

3 $$9x + 3y = 9$$
$$8x - 3y = 25$$

4 $$9x + 5y = 53$$
$$4y - 9x = 10$$

5 $$4x + 3y = 32$$
$$8y - 4x = 24$$

6 $$3(7x + 3y) = 189$$
$$9y - 12x = -9$$

C Find the value of the first variable

$$3x + 8y = 74 \ldots\ldots (i)$$
$$3x + y = 25 \ldots\ldots (ii)$$

Name the equations so we can refer to them easily

Subtract (i) – (ii)

We subtract because the x-terms (that we are trying to get rid of) are both the same sign (ie positive). Sometimes we do (ii) – (i)

$$(3-3)x + (8-1)y = 74 - 25$$
$$7y = 49$$
$$y = 7$$

Find the first variable with these equations

1 $$6x + 5y = 74$$
$$6x + 2y = 25$$

2 $$x + 4y = 20$$
$$3x + 4y = 36$$

3 $$5x + y = 37$$
$$3x + y = 23$$

D Find the value of the second variable

$$3x + 8y = 74 \ldots\ldots (i)$$

We have worked out that y=7 in part C.

Substitute in (i)

We take the value for y (which we found in part C) and substitute it into this equation.

$$3x + 8(7) = 74$$
$$3x = 74 - 56$$
$$3x = 18$$
$$x = 6$$

You can check your answer by substituting into equation (ii) to see if it works. If it does... well done

Now answer questions 1 – 7 given in **A**

Simultaneous Equations

E What if the coefficients are related but not the same?

$3x + 8y = 106$ …… (i)
$x + 2y = 28$ …… (ii)

Neither the x nor the y coefficients are the same in this pair of equations. You may notice that if you multiply equation (ii) × 4, you will get a y coefficient of 8 in both equations. You can then continue as before.

$3x + 8y = 106$ …… (i)
$x + 2y = 28$ …… (ii)

Multiply (ii) × 4

$4x + 8y = 112$ …… (iii)

Subtract (iii) – (i)

$(4-1)x + (8-8)y = 112 - 106$

Decide which equation you are going to multiply, and by how much

1. $6x + 5y = 119$
   $2x + 2y = 44$
2. $x + 9y = 179$
   $3x + 4y = 123$
3. $9x + y = 117$
   $3x - 4y = -39$

F What if the coefficients are unrelated and not the same?

$7x + 8y = 43$ …… (i)
$5x + 3y = 47$ …… (ii)

Here, the coefficients are unrelated: 8 is not a multiple of 3; 7 is not a multiple of 5. You decide which term, x or y, you wish to get rid of.
Imagine we have chosen, "Get rid of x".
Multiply equation (i) × 5 (which is the x coefficient in equation (ii)).
Then, multiply equation (ii) × 7 (which is the x coefficient in equation (i)).
If you had decided on y, you would multiply (i)× 3 and (ii) × 8.

Multiply (i) × 5, (ii) × 7

$35x + 40y = 215$ …… (iii)
$35x + 21y = 329$ …… (iv)

Subtract (iii) – (iv)

⋮

Find equations (iii) and (iv)

1. $7x + 8y = 43$
   $5x + 3y = 47$
2. $5x + 9y = -9$
   $3x + 4y = 3$
3. $8x + 7y = 67$
   $3x - 4y = 98$

G Solve the equations in their entirety.

$9x + 8y = 47$ …… (i)
$5x - 3y = 41$ …… (ii)

Multiply (i) × 5, (ii) × 9

$45x + 40y = 235$ …… (iii)
$45x - 27y = 369$ …… (iv)

Subtract (iii) – (iv)

$(45-45)x + (40-(-27))y = 235-369$
$67y = -134$
$y = \frac{-134}{67}$
$y = -2$

Be careful with double minuses etc.

Substitute in (i)

$9x + 8(-2) = 47$
$9x - 16 = 47$
$9x = 47 + 16$
$9x = 63$
$x = 7$

Check in (ii)

$5x - 3y = 41$ …… (ii) ✓
$5(7) - 3(-2) = 41$

1. $11x + 6y = 30$ …… (i)
   $5x - 5y = 60$ …… (ii)
2. $4x - 7y = -355$ …… (i)
   $8x - 5y = -161$ …… (ii)
3. $8x + 5y = 81$ …… (i)
   $2x - 5y = 89$ …… (ii)
4. $7x + 4y = 43$ …… (i)
   $9x + 3y = 21$ …… (ii)
5. $2x + 7y = 5.2$ …… (i)
   $12x - 4y = 12.8$ …… (ii)
6. $7x + 6y = 229$ …… (i)
   $9x - 4y = -303$ …… (ii)

Equal coefficients — This may involve finding one unknown in each equation with the same coefficients or multiplying one of more of the equations to manufacture the same coefficients.

Get rid of one unknown — This involves either adding (if the coefficients are of different signs (- or +) or subtracting if they are the same sign.

Substitute into equation (i) or (ii) — Swap the value you have found for x or the value you have found for y into equation (i) or (ii). Solve the equation.

Check your answer — Swap the value you have found for x or the value you have found for y into the other equation. If it works – give yourself a tick!!

Note the strategy for solving simultaneous linear equations

Answers to part G

1	x=6	y=-6
2	x=18	y=61
3	x=17	y=-11
4	x=-3	y=16
5	x=1.2	y=0.4
6	x=-11	y=51